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Penyelesaian PD dengan Deret Kuasa

3). Dapatkan PU dari PD $y' + y = \cos x$, $y(0) = 1$. berikan nilai numerik sampai dengan a_4 !

Anggap penyelesaiannya adalah $\sum_{n=0}^{\infty} a_n x^n = y$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=}^{\infty} n a_n x^{n-1}$$

$$= \sum_{n=0}^{\infty} n+1\, a_{n+1} x^n$$

•> Deret maclaurin $\cos x$

$$f(x) = f(0) + \frac{f'(0)}{1} x + \frac{f''(0)}{2!} x^2 + \frac{f'''(0)}{3!} x^3 + \frac{f^{(4)}(0)}{4!} x^4 + \dots$$

$$= 1 + \frac{(-\sin(0))}{1} x + \frac{-\cos(0)}{2!} \cdot x^2 + \frac{\sin(0)}{3!} x^3 + \frac{\cos(0)}{4!} x^4 + \dots$$

$$= 1 - \frac{1}{2!} x^2 + \frac{1}{4!} x^4$$

$\Rightarrow$ $y' + y = \cos x$

$$\sum_{n=0}^{\infty} (n+1)\, a_{n+1} x^n + \sum_{n=0}^{\infty} a_n x^n = 1 - \frac{1}{2!} x^2 + \frac{1}{4!} x^4$$

•> Saat $n=0$

$$a_1 x^0 + a_0 = 1$$

$$a_1 = 1 - a_0$$

•> Saat $n=1$

$$2a_2 + a_1 = 0$$

$$a_2 = -\frac{1}{2} a_1$$

$$= -\frac{1}{2}(1-a_0)$$

•> Saat $n=2$

$$3a_3 + a_2 = -\frac{1}{2}$$

$$3a_3 = -\frac{1}{2} - a_2$$

$$a_3 = -\frac{1}{3!} - \frac{a_2}{3}$$

$$= -\frac{1}{3!} + \frac{1}{3!}(1-a_0)$$

-> Saat $n = 3$

$$4a_4 + a_3 = 0$$
$$4a_4 = -a_3$$
$$a_4 = -\frac{1}{4}a_3$$
$$= \frac{1}{4!} - \frac{1}{4!}(1-a_0)$$

Sehingga, diperoleh :

$$y = \sum_{n=0}^{\infty} a_n x^n$$
$$= a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$$
$$= a_0 + (1-a_0)x + \left[-\frac{1}{2!}(1-a_0)\right]x^2 + \left[-\frac{1}{3!} + \frac{1}{3!}(1-a_0)\right]x^3 + \left[\frac{1}{4!} - \frac{1}{4!}(1-a_0)\right]x^4$$
$$= a_0 + x - a_0 x + \left[-\frac{1}{2!}(1-a_0)\right]x^2 + \left[-\frac{1}{3!} + \frac{1}{3!}(1-a_0)\right]x^3 + \left[\frac{1}{4!} - \frac{1}{4!}(1-a_0)\right]x^4$$
$$= a_0\left[1 - x + \frac{1}{2!}x^2 - \frac{1}{3!}x^3 + \frac{1}{4}x^4\right] + \left[x - \frac{1}{2!}x^2 + \left[-\frac{1}{3!} + \frac{1}{3!}\right]x^3 + \left[\frac{1}{4!} - \frac{1}{4!}\right]x^4\right.$$
$$= a_0\left[1 - x + \frac{1}{2!}x^2 - \frac{1}{3!}x^3 + \frac{1}{4}x^4\right] + \left[x - \frac{1}{2!}x^2 + \dots\right]$$

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5). Dapatkan PU dari PD $y' + (1-x)y = \frac{1}{2-x}$, $y(1) = 1$

- Untuk lebih mempermudah, misalkan $u = x - 1$, sehingga

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
$$= \frac{dy}{du}$$

- Maka PD dapat dituliskan

$$\frac{dy}{du} - uy = \frac{1}{2-(u+1)} \Rightarrow \frac{dy}{du} - u.y = \frac{1}{1-u}$$

Karena saat $x = 1$ memenuhi $y(1) = 1$, maka ketika $u = x - 1$, memenuhi

$$u = 1 - 1 = 0$$

▶ Deretkan $\frac{1}{1-u}$ di sekitar $u=0$

$= \bullet\ f'(u) = \frac{1}{(1-u)^2}$

$\bullet\ f''(u) = -\frac{2(1-u)(-1)}{(1-u)^4}$

$= \frac{2-2u}{(1-u)^4}$

$\bullet\ f'''(u) = \frac{-2(1-u)^4 - (2-2u).4(1-u)^3(-1)}{(1-u)^6}$

$= \frac{-2(1-u)^4 + 8(1-u)^4}{(1-u)^6}$

$= \frac{6}{(1-u)^2}$

$\bullet\ f^{(4)}(u) = \frac{-6.2(1-u)(-1)}{(1-u)^4}$

$= \frac{12-12u}{(1-u)^4}$

Deret maclaurin :

$$f(x) = f(0) + \frac{f'(0)}{1}x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4$$

$$= 1 + x + \frac{2}{2!}x^2 + \frac{6}{3!}x^3 +$$

$$= 1 + x + x^2 + x^3 + \dots$$

Anggap penyelesaian : $y = \sum_{n=0}^{\infty} a_n x^n$

$$y' = \sum_{n=}^{\infty} n a_n x^{n-1}$$

$$= \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n$$

Sehingga

$$\sum_{n=0}^{\infty} (n+1) a_{n+1} u^n - u . \sum_{n=0}^{\infty} a_n u^n = 1 + u + u^2 + u^3 + \dots$$

$$\sum_{n=0}^{\infty} (n+1) a_{n+1} u^n - \sum_{n=0}^{\infty} a_n u^{n+1} = 1 + u + u^2 + u^3 + \dots$$

$$\sum_{n=0}^{\infty} (n+1) a_{n+1} u^n - \sum_{n=1}^{\infty} a_{n-1} u^n = 1 + u + u^2 + u^3 + \dots$$

•> Saat $n=0$

$a_1 = 1$

•> Saat $n=1$

$2a_2 - a_0 = 1$

$a_2 = \frac{1+a_0}{2}$

•> Saat $n=2$

$3a_3 - a_1 = 1$

$a_3 = \frac{1+a_1}{3}$

$= \frac{2}{3}$

•> Saat $n=3$

$4a_4 - a_2 = 1$

$a_4 = \frac{1+a_2}{4}$

$a_4 = \frac{1+\left(\frac{1+a_0}{2}\right)}{4}$

$= \frac{3+a_0}{2.4}$

$= \frac{a_0+3}{8}$

> Saat $n=4 \rightarrow$ $5a_5 - a_3 = 1$

$5a_5 = 1 + a_3$

$5a_5 = 1 + \frac{2}{3}$

$5a_5 = \frac{5}{3}$

$a_5 = \frac{1}{3}$

Diperoleh : Penyelesaian $y = \sum_{n=0}^{\infty} a_n x^n$

PUPD

$$y = a_0 + 1.(u) + \left[\frac{1+a_0}{2}\right]u^2 + \frac{2}{3}u^3 + \left[\frac{a_0+3}{8}\right]u^4 + \frac{1}{3}u^5$$

$$y = a_0\left[1 + \frac{u^2}{2} + \frac{u^4}{8} + \ldots\right] + \left[u + \frac{u^2}{2} + \frac{2}{3}u^3 + \frac{3}{8}u^4 + \frac{1}{3}u^5 + \ldots\right]$$

Ubah kembali ke x [Ingat : $u = x-1$]

$$y = a_0\left[1 + \frac{(x-1)^2}{2} + \frac{(x-1)^4}{8} + \ldots\right] + \left[(x-1) + \frac{(x-1)^2}{2} + \frac{2}{3}(x-1)^3 + \frac{3}{8}(x-1)^4 + \ldots\right]$$

- Tentukan Penyelesaian Khusus

$y(1) = 1$

$$= a_0\left[1 + \frac{1}{2} + \frac{1}{8} + \ldots\right] + \left[1 + \frac{1}{2} + \frac{2}{3} + \frac{3}{8} + \ldots\right]$$

6). Selesaikan masalah PD berikut dengan Deret Kuasa

e). $y'' - y' + xy = 0$

misal penyelesaian : $y = \sum_{n=0}^{\infty} a_n x^n$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} = \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n$$

Sehingga, dapat ditulis :

$$\sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n - \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n + x \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n - \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n + \sum_{n=0}^{\infty} a_n x^{n+1} = 0$$

$$\sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n - \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n + \sum_{n=1}^{\infty} a_{n-1} x^n = 0$$

$$2a_2 - a_1 + \sum_{n=1}^{\infty} \left[(n+2)(n+1) a_{n+2} - (n+1) a_{n+1} + a_{n-1} \right] = 0$$

- $n = 0 \rightarrow a_2 = \frac{1}{2} a_1$
- $n = 1 \rightarrow 6a_3 - 2a_2 + a_0 = 0$

$$6a_3 = a_1 - a_0$$

$$a_3 = \frac{1}{6}(a_1 - a_0)$$

- $n = 2 \rightarrow 12a_4 - 3a_3 + a_1 = 0$

$$12a_4 = 3\left[\frac{1}{6}a_1 - \frac{1}{6}a_0\right] - a_1$$

$$12a_4 = \frac{1}{2}a_1 - \frac{1}{2}a_0 - a_1$$

$$a_4 = -\frac{1}{24}(a_1 + a_0)$$

- $n = 3 \rightarrow 20a_5 - 4a_4 + a_2 = 0$

$$20a_5 = -\frac{1}{6}(a_1 + a_0) - \frac{1}{2}a_1$$

$$a_5 = -\frac{1}{120}(a_1 + a_0) - \frac{1}{40}a_1$$

$$= -\frac{4}{120}a_1 - \frac{1}{120}a_0$$

PUPD :

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$= a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots$$

$$= a_0 + a_1 x + \frac{1}{2!} a_1 x^2 + \frac{1}{3!}(a_1 - a_0) x^3 - \frac{1}{4!}(a_1 + a_0) x^4 + \dots$$

$$= a_0 + a_1 x + \frac{1}{2!} a_1 x^2 + \frac{1}{3!} a_1 x^3 - \frac{1}{3!} a_0 x^3 - \frac{1}{4!} a_1 x^4 - \frac{1}{4!} a_0 x^4 + \dots$$

$$= \left[1 - \frac{1}{3!} x^3 - \frac{1}{4!} x^4 + \dots\right] a_0 + a_1 \left(x + \frac{1}{2!} x^2 + \frac{1}{3!} x^3 - \frac{1}{4!} x^4\right)$$

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